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$N = 4$ central charge superspace at work for supergravity coupled to an arbitrary number of abelian vector multiplets

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Abstract: We present the description in central charge superspace of $N = 4$ supergravity with antisymmetric tensor coupled to an arbitrary number of abelian vector multiplets. All the gauge vectors of the coupled system are treated on the same footing as gauge fields corresponding to translations along additional bosonic coordinates. It is the geometry of the antisymmetric tensor which singles out which combinations of these vectors belong to the supergravity multiplet and which are the additional coupled ones. Moreover, basic properties of Chapline-Manton coupling mechanism, as well as the $\frac{SO(6,n)}{SO(6)\times SO(n)}$ sigma model of the Yang-Mills scalars are found as arising from superspace geometry.

Keywords: extended supersymmetry, supergravity, central charge superspace, equations of motion.

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1. Introduction

Interest in $N = 4$ supergravity originates from the fact that it has the largest amount of supersymmetry that still allows existence of matter multiplets. The supergravity multiplet contains one graviton, 4 gravitini, 6 vectors, 4 spin 1/2 fermions, 1 scalar and 1 pseudoscalar, or, equivalently with dualization, one scalar and one antisymmetric 2-tensor. Matter comes only under the form of vector multiplets which contain 1 vector, 4 spinors, and 6 scalars.

$N = 4$ supergravity exists in two versions, one with global $SO(4)$ [1, 2] and one with global $SU(4)$ symmetries [3]. As long as these symmetries are not gauged, the two versions are equivalent [3] and we will be interested in the SU(4) theory whose advantage is to restrict non-polynomiality to exponentials of the scalar. The Lagrangian and supersymmetry transformations for pure supergravity were first obtained in [3]. Moreover, equivalence with the $SO(4)$ theory was shown, and the scalar and pseudoscalar of the theory were found to parameterize the group $SU(1,1)$. This constitutes the first case of appearance of hidden symmetries in supergravities. In [4] the pseudo-scalar was dualized to an antisymmetric tensor, giving the theory vanishing trace anomaly. Thus a relation with higher dimensional supergravities and string theory was suspected.

This relation was further explored in [5] where the gravity multiplet of $N = 1$ $d = 10$ supergravity was dimensionally reduced to obtain $N = 4$ supergravity coupled to 6 Yang-Mills vector multiplets. Upon reduction the metric and the tensor both give 6 vectors $(g_\mu)_i$ and $(B_\mu)_i$ where μ is a 4-dimensional space-time index and i is a 6-dimensional internal index. The "physical" vectors which are members of the multiplets are linear combinations of these

$$(\mathcal{A}_\mu)_i \; = \; (g_\mu)_i + (B_\mu)_i, \qquad (\mathcal{B}_\mu)_i \; = \; (g_\mu)_i - (B_\mu)_i \tag{1.1}$$

where $(\mathcal{A}_\mu)_i$ are the graviphotons and $(\mathcal{B}_\mu)_i$ are the Yang-Mills vectors. The 36 scalars g_{ij} and B_{ij} go to a non-linear sigma model and the non compact symmetry is extended to $SO(6,6) \times SU(1,1)$. One of the problems in this situation is that the couplings between the graviphotons and the Yang-Mills vectors are quite intricate, and a generalization to any number n of Yang-Mills multiplets was not obvious. This was later solved in [6] for abelian multiplets, and in [7], [8], [9] for non-abelian ones, using conformal methods or more specific reduction procedures.

However, a complete transcription of these results in superspace formalism is still lacking. Our purpose in the present paper is to attempt to (partially) fill in this gap. Among many advantages, superspace provides concise and elegant descriptions of the equivalent component theories, as well as a promising framework for discovery of auxiliary fields in the hope of finding off-shell formulations. $N = 4$ pure supergravity multiplet with an antisymmetric tensor (which we call N-T multiplet here) was first obtained in [4]. Its formulation in superspace encountered a number of problems identified in [10] and overcome in [11]. Recently in [12], the graviphotons were identified in the central charge sector allowing an elegant description of the Chern-Simons forms as coming from geometric considerations. The equivalence with the component formalism was proved in [13] where the equations of motion coming from the Bianchi Identities on superspace were shown to be identical to the ones derived from the Lagrangian of [4]. Considering the historical developments in the component formalism, the next "natural" step would be to look for n abelian Yang-Mills multiplets on the superspace. This is what we are presenting in this work. We show that the central charge sector used in [13] can be extended to describe n Yang-Mills vectors. In section 2.1 we recall the constraints allowing the identification of the gravity multiplet and we give the modifications we bring to the 0 dimensional scalar sector such that the central charge sector can accommodate the extra matter multiplets. $6n$ new scalars are found. In section 2.2, we explore the implications of these new constraints by giving the solution to the Bianchi Identities, and we identify the $4n$ spin 1/2 fermions belonging to the Yang-Mills multiplets. In section 2.3 we identify the Yang-Mills vectors. In particular, we show that it is the geometry of the 2-form, which, through the action of projectors, separates the supergravity vectors from the Yang-Mills ones. Finally in section 3 we indicate how these results are equivalent to the component formulation in [6]. Then we conclude in section 4 by discussing a few generalizations one can think of, such as describing

non-abelian Yang-Mills multiplets, or some hints for a possible off-shell formulation.

2. From pure supergravity to supergravity coupled to n vector multiplets

In this section we recall the essential results of [12] concerning the identification of the components of the N-T multiplet. Recall that in geometrical formulation of supergravity theories the basic dynamic variables are chosen to be the vielbein and the connection. Considering central charge superspace this framework provides a unified geometric identification of graviton, gravitini and graviphotons in the frame $E^{\mathcal{A}} = (E^a, E^\alpha_{\mathrm{A}}, E_{\dot\alpha}^{\mathrm{A}}, E^{\mathbf{u}})$

$$E^a\| \;=\; \mathrm{d}x^m e_m{}^a\ , \quad E^\alpha_{\mathrm{A}}\| \;=\; \frac{1}{2}\,\mathrm{d}x^m \psi_{m\mathrm{A}}^{\ \alpha}\ , \quad E_{\dot\alpha}^{\mathrm{A}}\| \;=\; \frac{1}{2}\,\mathrm{d}x^m \bar\psi_{m\dot\alpha}^{\ \mathrm{A}}\ , \quad E^{\mathbf{u}}\| \;=\; \mathrm{d}x^m v_m{}^{\mathbf{u}}\ . \tag{2.1}$$

Here a is the vectorial index, $\alpha, \dot\alpha$ are the spinorial indices, $\mathrm{A} = 1..4$ is the internal symmetry index, while $\mathbf{u} = 1..6+n$ counts the central charge coordinates. Moreover, the antisymmetric tensor can be identified in a superspace 2–form B:

$$B\| \;=\; \frac{1}{2}\,\mathrm{d}x^m \mathrm{d}x^n b_{nm}. \tag{2.2}$$

The remaining component fields, a real scalar and 4 helicity 1/2 fields, are identified in the supersymmetry transforms of the vielbein and 2–form, that is in torsion ($T^{\mathcal{A}} = DE^{\mathcal{A}}$) and 3–form ($H = dB$) components. The Bianchi identities satisfied by these objects are

$$DT^{\mathcal{A}} \;=\; E^{\mathcal{B}} R_{\mathcal{B}}{}^{\mathcal{A}}\ , \qquad\qquad \mathrm{d}H \;=\; 0\ , \tag{2.3}$$

and, displaying the form coefficients

$$\begin{aligned}
\left({}_{\mathcal{DCB}}{}^{\mathcal{A}}\right)_T \quad &: \quad E^{\mathcal{B}} E^{\mathcal{C}} E^{\mathcal{D}} \left(\mathcal{D}_{\mathcal{D}} T_{\mathcal{CB}}{}^{\mathcal{A}} + T_{\mathcal{DC}}{}^{\mathcal{F}} T_{\mathcal{FB}}{}^{\mathcal{A}} - R_{\mathcal{DCB}}{}^{\mathcal{A}}\right) = 0, && (2.4)\\
\left({}_{\mathcal{DCBA}}\right)_H \quad &: \quad E^{\mathcal{A}} E^{\mathcal{B}} E^{\mathcal{C}} E^{\mathcal{D}} \left(2\mathcal{D}_{\mathcal{D}} H_{\mathcal{CBA}} + 3 T_{\mathcal{DC}}{}^{\mathcal{F}} H_{\mathcal{FBA}}\right) = 0. && (2.5)
\end{aligned}$$

2.1 The constraints

The geometrical description of the N-T multiplet is based on a set of natural constraints in central charge superspace with structure group $SL(2,\mathbb{C}) \times U(4)$. The central charge sector is chosen to be trivial in the sense that the covariant derivative in the central charge direction $\mathcal{D}_{\mathbf{u}}$ vanishes on all superfields as well as the connection $\Phi_{\mathbf{z}}{}^{\mathbf{u}}$ is zero. Conventions for vector and spinor representations of the Lorentz group are those of [14].

The generalizations of the canonical dimension 0 "trivial constraints" [15] to central charge superspace are

$$T_{\gamma\beta}^{\mathrm{CB}a} \;=\; 0\ , \qquad T_{\gamma\mathrm{B}}^{\mathrm{C}\dot\beta a} \;=\; -2i\delta^{\mathrm{C}}_{\mathrm{B}}(\sigma^a\epsilon)_\gamma{}^{\dot\beta}\ , \qquad T^{\dot\gamma\dot\beta a}_{\mathrm{CB}} \;=\; 0\ , \tag{2.6}$$

$$T_{\gamma\beta}^{\mathrm{CB}\mathbf{u}} \;=\; 4\epsilon_{\gamma\beta} L^{1/2} \mathsf{t}^{[\mathrm{CB}]\mathbf{u}}\ , \qquad T_{\gamma\ \mathrm{B}}^{\mathrm{C}\dot\beta\mathbf{u}} \;=\; 0\ , \qquad T^{\dot\gamma\dot\beta\mathbf{u}}_{\mathrm{CB}} \;=\; 4\epsilon^{\dot\gamma\dot\beta} L^{1/2} \mathsf{t}_{[\mathrm{CB}]}{}^{\mathbf{u}}\ . \tag{2.7}$$

As explained in detail in the article [12], the soldering is achieved by requiring some analogous, "mirror"-constraints for the 2–form sector. Besides the -1/2 dimensional constraints $H_{\gamma\,\beta\alpha}^{\mathrm{CBA}} = H_{\gamma\,\beta\mathrm{A}}^{\mathrm{CB}\dot{\alpha}} = H_{\gamma\,\mathrm{BA}}^{\mathrm{C}\dot{\beta}\dot{\alpha}} = H_{\mathrm{CBA}}^{\dot{\gamma}\dot{\beta}\dot{\alpha}} = 0$, we impose

$$H_{\gamma\beta a}^{\mathrm{CB}} \;=\; 0\;, \qquad H_{\gamma\mathrm{B}a}^{\mathrm{C}\dot{\beta}} \;=\; -2i\delta_{\mathrm{B}}^{\mathrm{C}}(\sigma_a\epsilon)_{\gamma}{}^{\dot{\beta}}L\;, \qquad H_{\mathrm{CB}a}^{\dot{\gamma}\dot{\beta}} \;=\; 0\;, \tag{2.8}$$

$$H_{\beta\alpha\mathbf{u}}^{\mathrm{BA}} = 4\epsilon_{\beta\alpha}L^{1/2}\mathfrak{h}_{\mathbf{u}}{}^{[\mathrm{BA}]}\,, \qquad H_{\gamma\,\mathrm{B}\mathbf{u}}^{\mathrm{C}\dot{\beta}} = 0\,, \qquad H_{\mathrm{B\,A}\mathbf{u}}^{\dot{\beta}\dot{\alpha}} = 4\epsilon^{\dot{\beta}\dot{\alpha}}L^{1/2}\mathfrak{h}_{\mathbf{u}[\mathrm{BA}]}\,, \tag{2.9}$$

with L a real superfield. The physical scalar ϕ of the multiplet, called also graviscalar, is identified in this superfield, parameterized as $L = e^{2\phi}$. In turn, the helicity 1/2 gravigini fields are identified as usual [16], [17], [10] in the 1/2–dimensional torsion component

$$\epsilon^{\beta\gamma}T_{\gamma\,\beta\dot{\alpha}}^{\mathrm{CBA}} \;=\; 2T^{[\mathrm{CBA}]}{}_{\dot{\alpha}}, \qquad \epsilon_{\dot{\beta}\dot{\gamma}}T_{\mathrm{C\,BA}}^{\dot{\gamma}\dot{\beta}\alpha} \;=\; 2T_{[\mathrm{CBA}]}{}^{\alpha}. \tag{2.10}$$

The scalar, the four helicity 1/2 fields, together with the gauge fields defined in (2.1) and (2.2) constitute the N-T on-shell N=4 supergravity multiplet. Recall that in the case of pure supergravity [12, 13], the matrix elements $\mathfrak{t}^{[\mathrm{BA}]\mathbf{u}}$, $\mathfrak{t}_{[\mathrm{BA}]}{}^{\mathbf{u}}$, $\mathfrak{h}_{\mathbf{u}[\mathrm{BA}]}$, $\mathfrak{h}_{\mathbf{u}}{}^{[\mathrm{BA}]}$ are constrained to be covariantly constant under the structure group $SL(2,\mathbb{C})\otimes SU(4)$. However, by leaving them arbitrary, extra multiplets can be accommodated in the same geometry [10]. In particular, imposing the self–duality conditions

$$\mathfrak{t}^{[\mathrm{DC}]\mathbf{u}} \;=\; \frac{q}{2}\,\varepsilon^{\mathrm{DCBA}}\mathfrak{t}_{[\mathrm{BA}]}{}^{\mathbf{u}}\,, \qquad \text{with} \quad q = \pm 1 \tag{2.11}$$

we expect to describe a number of on-shell vector multiplets [18] coupled to the N-T multiplet. Recall, that the central charge index, $\mathbf{u}$, runs from 1 to $6+n$. Since the number of gauge vectors taking part of the N-T multiplet is 6, we expect to deal implicitly with n independent additional gauge vectors in the geometry, which take part of vector multiplets.

Like in the case of pure N-T supergravity, let us suppose the existence of a covariantly constant metric $\mathfrak{g}^{\mathbf{zu}}$

$$\mathfrak{g}^{\mathbf{zu}}\mathfrak{g}_{\mathbf{uv}} = \delta_{\mathbf{v}}^{\mathbf{z}}, \tag{2.12}$$

which connects the components of the 2-form $T^{\mathbf{u}}$ to the components of H having at least one central charge index

$$H_{\mathbf{z}\mathcal{D}\mathcal{C}} \;=\; T_{\mathcal{D}\mathcal{C}}{}^{\mathbf{u}}\mathfrak{g}_{\mathbf{uz}}. \tag{2.13}$$

However, unlike the pure supergravity case, this metric is not entirely given as a function of the Lorentz scalars $\mathfrak{t}^{[\mathrm{BA}]\mathbf{u}}$ or $\mathfrak{h}_{\mathbf{u}}{}^{[\mathrm{BA}]}$, which are in this case $6\times(6+n)$ matrices having at most rank 6.

One can further eliminate a big number of superfluous fields by assuming the constraint

$$T_{\mathbf{z}\mathcal{B}}{}^{\mathcal{A}} = 0, \tag{2.14}$$

as well as all possible compatible conventional constraints. Also, the constraints

$$\mathcal{D}_\alpha^{\mathrm{E}}(T^{[\mathrm{DC}]\mathbf{u}})H_{\mathbf{u}[\mathrm{BA}]} \;=\; 0\,, \qquad\qquad \mathcal{D}^{\dot\alpha}_{\mathrm{E}}(T_{[\mathrm{DC}]\mathbf{u}})H^{\mathbf{u}[\mathrm{BA}]} \;=\; 0 \tag{2.15}$$

at dimension 1/2 as well as

$$\bar\sigma^{b\dot\alpha\gamma}T_{\gamma}^{\mathrm{C}}{}_{b\dot\alpha}^{\mathrm{A}} = 0\,, \qquad\qquad \sigma^b_{\alpha\dot\gamma}T^{\dot\gamma}_{\mathrm{C}}{}_{b\mathrm{A}}^{\alpha} = 0\,, \tag{2.16}$$

at dimension 1 are used to put the gravity part on–shell.

Finally, in order to make possible the direct comparison with the results of pure N-T supergravity, let us define a covariant derivative $\hat{\mathcal{D}}$ under $SL(2,\mathbb{C}) \times SU(4)$,

$$\mathcal{D}v^{\mathrm{A}} \;=\; \hat{\mathcal{D}}v^{\mathrm{A}} - \chi^{\mathrm{A}}{}_{\mathrm{B}}v^{\mathrm{B}}, \tag{2.17}$$

where the shift $\chi^{\mathrm{A}}{}_{\mathrm{B}}$ in the connection is determined by the requirement

$$\hat{\mathcal{D}}(\mathfrak{t}^{[\mathrm{DC}]\mathbf{u}})\mathfrak{h}_{\mathbf{u}[\mathrm{BA}]} \;=\; 0. \tag{2.18}$$

2.2 Solution of the Bianchi identities

Now let us consider the torsion and 3–form H subject to the above summarized constraints, and look at the Bianchi Identities (2.4) and (2.5) as equations for the remaining components of these two objects. The solution of these Bianchi Identities will be presented in the order of growing canonical dimension.

The lowest canonical dimension Bianchi Identities are those with only spinorial indices written for the 3–form. Given the above constraints, they are satisfied if and only if the Lorentz scalars at dimension 0 satisfy

$$\mathfrak{g}_{\mathbf{uz}}\mathfrak{t}^{[\mathrm{DC}]\mathbf{u}}\mathfrak{t}_{[\mathrm{BA}]}{}^{\mathbf{z}} \;=\; \frac{1}{2}\delta^{\mathrm{DC}}_{\mathrm{BA}}. \tag{2.19}$$

One may recognize that they represent 6×6 equations for the $6\times(6+n)$ a priori independent scalar superfields $\mathfrak{t}^{[\mathrm{DC}]\mathbf{u}}$. This means, that there are $6 \times n$ degrees of freedom left in these scalars, which is exactly the number of scalars in n additional vector multiplets.

At dimension 1/2 the spinorial components of χ are obtained and they are found to be the same as in the pure N-T case

$$\chi_\alpha^{\mathrm{AB}}{}_{\mathrm{C}} \;=\; \frac{1}{4}\delta^{\mathrm{B}}_{\mathrm{C}}\chi^{\mathrm{A}}_\alpha, \qquad \chi^{\dot\alpha\mathrm{B}}_{\mathrm{A}}{}_{\mathrm{C}} \;=\; -\frac{1}{4}\delta^{\mathrm{B}}_{\mathrm{C}}\chi^{\dot\alpha}_{\mathrm{A}}, \tag{2.20}$$

where we used the notation $\chi^{\mathrm{A}}_\alpha = L^{-1}\mathcal{D}^{\mathrm{A}}_\alpha L$, $\chi^{\dot\alpha}_{\mathrm{A}} = L^{-1}\mathcal{D}^{\dot\alpha}_{\mathrm{A}} L$.

Then the solution of the Bianchi Identities at dimension 1/2 can be written in the following way

$$T_{[\mathrm{CBA}]\alpha} = q\varepsilon_{\mathrm{CBAF}}\chi^{\mathrm{F}}_\alpha \tag{2.21}$$

$$\hat{\mathcal{D}}^{\mathrm{D}}_\delta\mathfrak{t}^{[\mathrm{CB}]\mathbf{u}} = q\varepsilon^{\mathrm{DCBA}}\mathfrak{s}_{\delta\mathrm{A}}{}^{\mathbf{u}} \tag{2.22}$$

$$T^{\dot\gamma}_{\mathrm{C}\,b}{}^{\mathbf{u}} = \mathrm{i}(\bar\sigma_b)^{\dot\gamma\alpha}\left(\mathfrak{s}_{\alpha\mathrm{A}}{}^{\mathbf{u}} + \chi^{\mathrm{F}}_\alpha\mathfrak{t}_{[\mathrm{FA}]}{}^{\mathbf{u}}\right)L^{1/2} \tag{2.23}$$

while similar relations are implied for H which can be easily obtained using (2.13)

$$\hat{\mathcal{D}}^{\mathrm{D}}_{\delta}\mathfrak{h}_{\mathbf{u}}{}^{[\mathrm{CB}]} = q\varepsilon^{\mathrm{DCBA}}\mathfrak{g}_{\mathbf{uz}}\mathfrak{s}_{\delta\mathrm{A}}{}^{\mathbf{z}} \tag{2.24}$$

$$H^{\dot\gamma}_{\mathrm{C}b\mathbf{u}} = \mathrm{i}(\bar\sigma_b)^{\dot\gamma\alpha}\mathfrak{g}_{\mathbf{uz}}\left(\mathfrak{s}_{\alpha\mathrm{A}}{}^{\mathbf{z}} + \chi^{\mathrm{F}}_{\alpha}\mathfrak{t}_{[\mathrm{FA}]}{}^{\mathbf{z}}\right)L^{1/2}. \tag{2.25}$$

Inspecting these results one identifies the spinor of the supergravity multiplet in $T_{[\mathrm{CBA}]\alpha}$ as the lowest superfield component of $\chi^{\mathrm{A}}_{\alpha}$, while the spinor $\mathfrak{s}_{\alpha\mathrm{A}}{}^{\mathbf{u}}$ appearing in the spinorial derivative of the scalars has to take part of the n additional vector multiplets. Their number is $4*(6+n) - 4*6 = 4*n$, where the number of 4*6 independent relations in the spinorial component of (2.18) is considered.

The results at dimension 1 can be gathered in three sectors depending on the irreducible representation of the Lorentz group in which the double spinorial derivatives on the 0 dimensional scalar sits.

Let us start with the sector of the scalars and of the dual fieldstrength $H^*_a = \frac{1}{3!}\varepsilon_{abcd}H^{bcd}$ of the antisymmetric tensor. This sector is determined by the mixed derivatives on the 0 dimensional scalars:

$$K_{\beta}{}^{\dot\alpha\mathrm{B}}{}_{\mathrm{A}} \;=\; [\mathcal{D}^{\mathrm{B}}_{\beta}, \mathcal{D}^{\dot\alpha}_{\mathrm{A}}]L, \qquad \mathcal{D}^{\mathrm{D}}_{\delta}\mathfrak{s}^{\mathrm{C}\dot\beta\mathbf{u}}. \tag{2.26}$$

The Bianchi Identity $\left({}^{\mathrm{D}\dot\gamma}_{\delta\,\mathrm{C}ba}\right)_H$ is satisfied if and only if

$$\frac{1}{2}K_a{}^{\mathrm{D}}{}_{\mathrm{C}} + \delta^{\mathrm{D}}_{\mathrm{C}}H^*_a + 4\mathrm{i}U_a{}^{\mathrm{D}}{}_{\mathrm{C}}L \;=\; \mathfrak{g}_{\mathbf{zu}}\mathfrak{s}_{\mathrm{C}}{}^{\mathbf{z}}\sigma_a\mathfrak{s}^{\mathrm{D}\mathbf{u}}L + \frac{1}{2}\delta^{\mathrm{DE}}_{\mathrm{CF}}\chi^{\mathrm{F}}\sigma_a\chi_{\mathrm{E}}L. \tag{2.27}$$

This equation relates the commutator on L to the dual fieldstrength H^*_a of the antisymmetric tensor and the superfield $U_a{}^{\mathrm{D}}{}_{\mathrm{C}}$, which appears in the dimension 1 torsion components

$$T^{\mathrm{C}}_{\gamma}{}_{b}{}^{\alpha}_{\mathrm{A}} \;=\; -2(\sigma_{ba})_{\gamma}{}^{\alpha}U^{a\mathrm{C}}{}_{\mathrm{A}} \tag{2.28}$$

$$T^{\dot\gamma}_{\mathrm{C}}{}_{b}{}^{\mathrm{A}}_{\dot\alpha} \;=\; 2(\bar\sigma_{ba})^{\dot\gamma}{}_{\dot\alpha}U^{a\mathrm{A}}{}_{\mathrm{C}}. \tag{2.29}$$

Also, the Bianchi Identity $\left({}^{\mathrm{D}\dot\gamma\mathrm{B}\alpha}_{\delta\,\mathrm{C}\beta\mathrm{A}}\right)_T$ together with the definition of the $SU(4)$ covariant derivative gives an other independent relation for the commutator on L

$$K_a{}^{\mathrm{D}}{}_{\mathrm{C}} \;=\; 2\mathrm{i}\delta^{\mathrm{D}}_{\mathrm{C}}(U_a + \chi_a)L - (\delta^{\mathrm{DA}}_{\mathrm{BC}} + \delta^{\mathrm{D}}_{\mathrm{B}}\delta^{\mathrm{A}}_{\mathrm{C}})\chi^{\mathrm{B}}\sigma_a\chi_{\mathrm{A}}L. \tag{2.30}$$

The last Bianchi Identities relevant for this sector are carrying a central charge index, $\left({}^{\mathrm{DC}}_{\delta\,\gamma}{}_{b}{}^{\mathbf{u}}\right)_T$ and $\left({}^{\mathrm{DC}}_{\delta\,\gamma}{}_{b}{}^{\mathbf{u}}\right)_H$. Using (2.18), these imply the equations

$$K_a \;=\; 8\mathrm{i}\chi_a L + \chi^{\mathrm{A}}\sigma_a\chi_{\mathrm{A}}L - 2\mathfrak{g}_{\mathbf{zu}}\mathfrak{s}_{\mathrm{A}}{}^{\mathbf{z}}\sigma_a\mathfrak{s}^{\mathrm{A}\mathbf{u}}L \tag{2.31}$$

$$\widetilde{\chi}_a{}^{\mathrm{B}}{}_{\mathrm{A}} \;=\; -\frac{\mathrm{i}}{8}\left(\widetilde{\chi^{\mathrm{B}}\sigma_a\chi_{\mathrm{A}}} + 2\mathfrak{g}_{\mathbf{zu}}\widetilde{\mathfrak{s}_{\mathrm{A}}{}^{\mathbf{z}}\sigma_a\mathfrak{s}^{\mathrm{B}\mathbf{u}}}\right) \tag{2.32}$$

where no indices are written for the trace parts and the tilde denotes the traceless parts.

It is sufficient then to solve the system of equations for the trace part

$$U_a = \frac{\mathrm{i}}{8}\left(\chi^{\mathrm{A}}\sigma_a\chi_{\mathrm{A}} + 2\mathfrak{g}_{\mathbf{zu}}\mathfrak{s}_{\mathrm{A}}{}^{\mathbf{z}}\sigma_a\mathfrak{s}^{\mathrm{A}\mathbf{u}}\right) \tag{2.33}$$

$$K_a = -8H_a^* + 4\chi^{\mathrm{A}}\sigma_a\chi_{\mathrm{A}}L + 4\mathfrak{g}_{\mathbf{zu}}\mathfrak{s}_{\mathrm{A}}{}^{\mathbf{z}}\sigma_a\mathfrak{s}^{\mathrm{A}\mathbf{u}}L \tag{2.34}$$

$$\chi_a = \mathrm{i}H_a^*L^{-1} - \frac{3\mathrm{i}}{8}\left(\chi^{\mathrm{A}}\sigma_a\chi_{\mathrm{A}} + 2\mathfrak{g}_{\mathbf{zu}}\mathfrak{s}_{\mathrm{A}}{}^{\mathbf{z}}\sigma_a\mathfrak{s}^{\mathrm{A}\mathbf{u}}\right) \tag{2.35}$$

and for the traceless part

$$\widetilde{U}_a{}^{\mathrm{B}}{}_{\mathrm{A}} = \widetilde{\chi}_a{}^{\mathrm{B}}{}_{\mathrm{A}} = -\frac{\mathrm{i}}{8}\left(\widetilde{\chi^{\mathrm{B}}\sigma_a\chi_{\mathrm{A}}} + 2\mathfrak{g}_{\mathbf{zu}}\widetilde{\mathfrak{s}_{\mathrm{A}}{}^{\mathbf{z}}\sigma_a\mathfrak{s}^{\mathrm{B}\mathbf{u}}}\right) \tag{2.36}$$

$$\widetilde{K}_a{}^{\mathrm{B}}{}_{\mathrm{A}} = -2\widetilde{\chi^{\mathrm{B}}\sigma_a\chi_{\mathrm{A}}}L \tag{2.37}$$

in order to have all the objects of this sector expressed in terms of the dual fieldstrengths H_a^* and nonlinear terms in the spinors χ of the gravity and $\mathfrak{s}$ of the vector multiplets.

Notice, that the vector P_a which appears naturally in the superspace geometry

$$\mathcal{D}_\delta^{\mathrm{D}}T^{[\mathrm{CBA}]\dot{\alpha}} = q\varepsilon^{\mathrm{DCBA}}P_\delta{}^{\dot{\alpha}} \tag{2.38}$$

is also corrected by non-linear terms in the $\mathfrak{s}$ spinors of the additional vector multiplets

$$P_a = \mathrm{i}L^{-1}\mathcal{D}_aL + L^{-1}H_a^* - \frac{3}{4}\chi^{\mathrm{A}}\sigma_a\chi_{\mathrm{A}} - \frac{1}{2}\mathfrak{g}_{\mathbf{zu}}\mathfrak{s}_{\mathrm{A}}{}^{\mathbf{z}}\sigma_a\mathfrak{s}^{\mathrm{A}\mathbf{u}} \tag{2.39}$$

Finally, using all these results, one obtains for the mixed derivatives of $\mathfrak{s}$

$$\hat{\mathcal{D}}_\delta^{\mathrm{D}}\mathfrak{s}^{\mathrm{C}\dot{\beta}\mathbf{u}} = -2\mathrm{i}\hat{\mathcal{D}}_\delta{}^{\dot{\beta}}\mathfrak{t}^{[\mathrm{DC}]\mathbf{u}} + 2\mathfrak{g}_{\mathbf{zv}}\mathfrak{s}_{\delta\mathrm{F}}{}^{\mathbf{z}}\mathfrak{s}^{\mathrm{C}\dot{\beta}\mathbf{v}}\mathfrak{t}^{[\mathrm{FD}]\mathbf{u}} - \frac{1}{4}\chi_\delta^{\mathrm{D}}\mathfrak{s}^{\mathrm{C}\dot{\beta}\mathbf{u}}. \tag{2.40}$$

The second sector at dimension 1 is the sector of the gauge vectors $v_m{}^{\mathbf{u}}$, with fieldstrengths identified in $F_{ba}{}^{\mathbf{u}} \doteq T_{ba}{}^{\mathbf{u}}$. This sector is governed by the double derivatives which are symmetric in their spinorial indices

$$\mathcal{D}_{(\beta}^{\mathrm{B}}\mathcal{D}_{\alpha)}^{\mathrm{A}}L, \qquad \mathcal{D}_{(\delta}^{\mathrm{D}}\mathfrak{s}_{\mathrm{C}\beta)}{}^{\mathbf{u}}. \tag{2.41}$$

After an analysis of the relevant Bianchi Identities one finds

$$\mathcal{D}_{(\beta}^{\mathrm{B}}\mathcal{D}_{\alpha)}^{\mathrm{A}}L = -4F_{(\beta\alpha)}{}^{\mathbf{u}}\mathfrak{h}_{\mathbf{u}}{}^{[\mathrm{BA}]}L^{1/2} + qL\epsilon^{\mathrm{BAEF}}\mathfrak{g}_{\mathbf{uz}}\mathfrak{s}^{\mathbf{u}}_{(\beta\mathrm{E}}\mathfrak{s}^{\mathbf{z}}_{\alpha)\mathrm{F}} \tag{2.42}$$

$$G_{(\beta\alpha)[\mathrm{BA}]} = -2\mathrm{i}F_{(\beta\alpha)}{}^{\mathbf{u}}\mathfrak{h}_{\mathbf{u}[\mathrm{BA}]}L^{-1/2} + \mathrm{i}\mathfrak{g}_{\mathbf{uz}}\mathfrak{s}^{\mathbf{u}}_{(\beta\mathrm{B}}\mathfrak{s}^{\mathbf{z}}_{\alpha)\mathrm{A}} \tag{2.43}$$

$$G_{(\beta\alpha)}{}^{[\mathrm{BA}]} = -\mathrm{i}\frac{q}{2}\varepsilon^{\mathrm{BAEF}}\mathfrak{g}_{\mathbf{uz}}\mathfrak{s}^{\mathbf{u}}_{(\beta\mathrm{E}}\mathfrak{s}^{\mathbf{z}}_{\alpha)\mathrm{F}} \tag{2.44}$$

where $G_{(\beta\alpha)[\mathrm{BA}]}$ and $G_{(\beta\alpha)}{}^{[\mathrm{BA}]}$ are the selfdual parts of the antisymmetric tensors $G_{ba[\mathrm{BA}]}$ and $G_{ba}{}^{[\mathrm{BA}]}$ appearing in the torsion components

$$T_{\mathrm{C}}^{\dot{\gamma}}{}_{b}{}^{\alpha}_{\mathrm{A}} = \frac{1}{2}(\bar{\sigma}^f)^{\dot{\gamma}\alpha}G_{bf[\mathrm{CA}]} \tag{2.45}$$

$$T_\gamma^{\mathrm{C}}{}_{b}{}^{\mathrm{A}}_{\dot{\alpha}} = \frac{1}{2}(\sigma^f)_{\gamma\dot{\alpha}}G_{bf}{}^{[\mathrm{CA}]}. \tag{2.46}$$

Then the derivative of the spinor $\mathfrak{s}$ becomes

$$\hat{\mathcal{D}}^{\mathrm{D}}_{(\delta}\mathfrak{s}_{\beta)\mathrm{C}}{}^{\mathbf{u}} = 2\delta^{\mathrm{D}}_{\mathrm{C}}\left(F_{(\delta\beta)}{}^{\mathbf{u}} - F_{(\delta\beta)}{}^{\mathbf{z}}\mathfrak{h}_{\mathbf{z}[\mathrm{BA}]}\mathfrak{t}^{[\mathrm{BA}]\mathbf{u}}\right)L^{-1/2}$$
$$+2\mathfrak{g}_{\mathbf{vz}}\mathfrak{s}^{\mathbf{v}}_{(\delta\mathrm{B}}\mathfrak{s}^{\mathbf{z}}_{\beta)\mathrm{C}}\mathfrak{t}^{[\mathrm{BD}]\mathbf{u}} + \frac{1}{4}\chi^{\mathrm{D}}_{(\delta}\mathfrak{s}^{\mathbf{u}}_{\beta)\mathrm{C}} - \delta^{\mathrm{D}}_{\mathrm{C}}\chi^{\mathrm{F}}_{(\delta}\mathfrak{s}^{\mathbf{u}}_{\beta)\mathrm{F}}\,. \tag{2.47}$$

The last sector might be called the sector of auxiliary fields, which in the present on-shell case does not contain any new superfield. It is governed by the antisymmetric part of the double derivatives

$$\mathcal{D}^{\mathrm{A}\alpha}\mathcal{D}^{\mathrm{B}}_{\alpha}L, \qquad \mathcal{D}^{\mathrm{A}\alpha}\mathfrak{s}_{\alpha\mathrm{B}}{}^{\mathbf{u}}. \tag{2.48}$$

Here one finds the relations

$$\mathcal{D}^{\mathrm{A}\alpha}\mathcal{D}^{\mathrm{B}}_{\alpha}L = -2\mathfrak{g}_{\mathbf{zu}}\mathfrak{s}^{\mathrm{A}\mathbf{z}}\mathfrak{s}^{\mathrm{B}\mathbf{u}}L \tag{2.49}$$
$$\hat{\mathcal{D}}^{\mathrm{A}\alpha}\mathfrak{s}_{\alpha\mathrm{B}}{}^{\mathbf{u}} = 2\mathfrak{g}_{\mathbf{vz}}\mathfrak{s}_{\mathrm{F}}{}^{\mathbf{v}}\mathfrak{s}_{\mathrm{B}}{}^{\mathbf{z}}\mathfrak{t}^{[\mathrm{FA}]\mathbf{u}} + \frac{1}{4}\chi^{\mathrm{A}}\mathfrak{s}_{\mathrm{B}}{}^{\mathbf{u}} - 2\chi_{\mathrm{B}}\mathfrak{s}^{\mathrm{A}\mathbf{u}} + \delta^{\mathrm{A}}_{\mathrm{B}}\chi_{\mathrm{F}}\mathfrak{s}^{\mathrm{F}\mathbf{u}} \tag{2.50}$$

as well as that the central charge component of the shift in the connection, $\chi_{\mathbf{u}}{}^{\mathrm{B}}{}_{\mathrm{A}}$, has to vanish.

2.3 Identification of the component fields

To sum up, let us review the main results obtained so far. The identification of the N-T supergravity multiplet in the geometry goes exactly in the same manner as in the pure supergravity case [12, 13]. The difference arises from the fact that whereas the Lorentz scalars $\mathfrak{t}^{[\mathrm{BA}]\mathbf{u}}$ sitting in the 0 dimensional torsion components in the central charge direction are covariantly constant for the pure supergravity case, they are taken to be a priori general superfields here. It turns out that they belong to the only matter multiplets available in $N = 4$ supergravity, that are vector multiplets. Considering that they obey the 6×6 independent relations (2.19), we obtain $6 \times n$ scalars in n vector multiplets. Remark that (2.19) is precisely the relation obtained in [6] as a condition for breaking conformal symmetry and going to Poincaré gauge. However, notice, that in our approach the equation (2.19) satisfied by the scalars is a consequence of the Bianchi Identities of a 3–form H constrained in an analogous way as we did for the pure N-T supergravity. In particular, the constraint (2.13), which relates components of H to those of the torsion by a metric plays a key role in all the basic features of the coupled system.

As usual in the superspace formalism, supersymmetry generators are implemented as spinorial covariant derivatives. Therefore we expect to see the fermionic supersymmetric partners of the scalars, $\mathfrak{s}_{\alpha\mathrm{A}}{}^{\mathbf{u}}$, arising in the object $\hat{\mathcal{D}}^{\mathrm{D}}_{\alpha}\mathfrak{t}^{[\mathrm{CB}]\mathbf{u}}$. This is indeed the case as shown by equation (2.22), whereas the constraint (2.18) insures that only the right number, $n \times 4$, of them are non-vanishing. Further applying spinorial derivatives to the spinors $\mathfrak{s}_{\alpha\mathrm{A}}{}^{\mathbf{u}}$, we should see the fieldstrength of Yang-Mills vectors. Accordingly, (2.47) suggests to define

$$F^{\mathrm{YM}\mathbf{u}}_{ba} = F_{ba}{}^{\mathbf{z}}\left(\delta^{\mathbf{u}}_{\mathbf{z}} - \mathfrak{h}_{\mathbf{z}[\mathrm{BA}]}\mathfrak{t}^{[\mathrm{BA}]\mathbf{u}}\right). \tag{2.51}$$

Also, let us identify the gauge vectors of the supergravity multiplet, the graviphotons, as those which appear in the supersymmetry transformation of the spinor χ which takes part of the supergravity multiplet (2.42):

$$F_{ba}^{\mathrm{SG}\,\mathbf{u}} = F_{ba}{}^{\mathbf{z}} \mathfrak{h}_{\mathbf{z}[\mathrm{BA}]} \mathfrak{t}^{[\mathrm{BA}]\mathbf{u}}. \tag{2.52}$$

One might notice that the two definitions, (2.51) and (2.52), involve projectors on the fields which belong to the supergravity or the gauge multiplets:

$$P^{\mathrm{SG}}{}_{\mathbf{z}}{}^{\mathbf{u}} = \mathfrak{h}_{\mathbf{z}[\mathrm{BA}]} \mathfrak{t}^{[\mathrm{BA}]\mathbf{u}}, \qquad P^{\mathrm{YM}}{}_{\mathbf{z}}{}^{\mathbf{u}} = \delta_{\mathbf{z}}^{\mathbf{u}} - P^{\mathrm{SG}}{}_{\mathbf{z}}{}^{\mathbf{u}}. \tag{2.53}$$

Due to the identity (2.19) they possess the standard properties of projectors

$$(P^{\mathrm{SG}})^2 = P^{\mathrm{SG}}, \qquad (P^{\mathrm{YM}})^2 = P^{\mathrm{YM}}, \qquad P^{\mathrm{SG}} P^{\mathrm{YM}} = 0. \tag{2.54}$$

The dimension of the spaces on which they project can also be computed and found to be as expected

$$\mathrm{tr} P^{\mathrm{SG}} = 6 \qquad \mathrm{tr} P^{\mathrm{YM}} = n. \tag{2.55}$$

Let us then summarize the identification of the fields in the following table:

σ	N-T sugra multiplet	n gauge multiplets
2	$e_m{}^a$	
3/2	$\psi_{m}{}^{\alpha}_{\mathrm{A}}$	
1	$v_m^{\mathrm{SG}\,\mathbf{u}}$, $\quad F_{ba}^{\mathrm{SG}\,\mathbf{u}} P_{\mathbf{u}}^{\mathrm{YM}\,\mathbf{z}}\vert = 0$	$v_m^{\mathrm{YM}\,\mathbf{u}}$, $\quad F_{ba}^{\mathrm{YM}\,\mathbf{u}} P_{\mathbf{u}}^{\mathrm{SG}\,\mathbf{z}}\vert = 0$
1/2	$\chi_\alpha^{\mathrm{A}}\vert$	$\mathfrak{s}_{\alpha\mathrm{A}}{}^{\mathbf{u}}\vert$, $\quad \mathfrak{s}_{\alpha\mathrm{A}}{}^{\mathbf{u}} P_{\mathbf{u}}^{\mathrm{SG}\,\mathbf{z}}\vert = 0$
0_s+0_t	$L\vert$ and b_{mn}	$\mathfrak{t}^{[\mathrm{BA}]\mathbf{u}}\vert$, $\quad \mathfrak{g}_{\mathbf{uz}} \mathfrak{t}^{[\mathrm{DC}]\mathbf{u}} \mathfrak{t}_{[\mathrm{BA}]}{}^{\mathbf{z}}\vert = \frac{1}{2}\delta_{\mathrm{BA}}^{\mathrm{DC}}$

3. Discussion

To give further arguments for the equivalence of this formulation with the component formalism, we discuss now the modifications found in the equations of motion due to the emergence of the Yang-Mills sector. One of the most intriguing features of the above results is the correction of the antisymmetric part of the double derivative on L (2.49) by

the quadratic term in the gaugini. This term appears in the corresponding derivative of the gravigino superfield

$$\begin{aligned}\mathcal{D}^{\mathrm{D}\alpha}T_{[\mathrm{CBA}]\alpha} &= q\varepsilon_{\mathrm{CBAF}}L^{-1}\mathcal{D}^{\beta\mathrm{D}}\mathcal{D}^{\mathrm{F}}_{\beta}L \\ &= -2q\varepsilon_{\mathrm{CBAF}}\mathfrak{g}_{\mathbf{zu}}\mathfrak{s}^{\mathrm{D}\mathbf{z}}\mathfrak{s}^{\mathrm{F}\mathbf{u}}L.\end{aligned} \tag{3.1}$$

It is well known that the object $\mathcal{D}^{\mathrm{D}\alpha}T_{[\mathrm{CBA}]\alpha}$ is playing the role of an auxiliary field[1] in four dimensional $N=4$ conformal supergravity [17], [20]. It was explicitly shown in [13] how its vanishing implies the Dirac equation for the gravigino and at dimension 2 the equations of motion for the antisymmetric tensor and that of the scalar.

Let us do the exercise of deriving equations of motion in an analogous way as in the pure supergravity case [13]. However, this time we do not consider all the non-linear terms, but concentrate only on the main features of the coupling keeping only terms involving the fieldstrength of gauge vectors. In particular, using (2.38) one can write the identity

$$\sum_{\mathrm{DC}}\left(\left\{\mathcal{D}^{\mathrm{E}}_{\varepsilon},\mathcal{D}^{\dot\delta}_{\mathrm{D}}\right\}T_{[\mathrm{CBA}]\alpha}-\mathcal{D}^{\dot\delta}_{\mathrm{D}}\left(\mathcal{D}^{\mathrm{E}}_{\varepsilon}T_{[\mathrm{CBA}]\alpha}\right)\right) \;=\; 0. \tag{3.2}$$

Observe that the antisymmetric part of this relation in the indices ε and α gives rise to Dirac equation for the helicity 1/2 fields, that is $\partial^{\alpha\dot\delta}T_{[\mathrm{CBA}]\alpha}=0$ in the linear approach. However, this time $\mathcal{D}^{\mathrm{E}\alpha}T_{[\mathrm{CBA}]\alpha}$ is different from zero and gives rise to a fieldstrength of the Yang-Mills vectors by (2.47) when the spinor derivative acts on the spinors $\mathfrak{s}$

$$\partial^{\alpha\dot\delta}\chi^{\mathrm{A}}_{\alpha} \;=\; 2\mathrm{i}F^{(\dot\delta\dot\alpha)\mathbf{v}}\mathfrak{s}^{\mathrm{A}\mathbf{u}}_{\dot\alpha}P^{\mathrm{YM}}{}_{\mathbf{v}}{}^{\mathbf{z}}\mathfrak{g}_{\mathbf{zu}}L^{-1/2}+... \tag{3.3}$$

After further differentiating by $\mathcal{D}_{\dot\delta\mathrm{A}}$, in order to obtain the expression of $\Box L$ one will need to use the algebra of derivatives on superspace. This operation (intimately connected to the gravity part) involves torsion components of (2.45) near the spinorial derivative on $\chi^{\mathrm{A}}_{\alpha}$ (2.42), both containing the vectors of the supergravity multiplet:

$$\Box L \;=\; -\mathrm{i}\partial^a H^*_a+2\left(F^{(\dot\alpha\dot\beta)\mathbf{u}}F_{(\dot\alpha\dot\beta)}{}^{\mathbf{z}}P^{\mathrm{YM}}{}_{\mathbf{u}}{}^{\mathbf{v}}-F^{(\alpha\beta)\mathbf{u}}F_{(\alpha\beta)}{}^{\mathbf{z}}P^{\mathrm{SG}}{}_{\mathbf{u}}{}^{\mathbf{v}}\right)\mathfrak{g}_{\mathbf{vz}}+... \tag{3.4}$$

On the one hand, taking the real part of this relation one finds the equation of motion for the scalar

$$\Box L \;=\; -\frac{1}{2}F^{ba\mathbf{u}}F_{ba}{}^{\mathbf{z}}\left(P^{\mathrm{SG}}-P^{\mathrm{YM}}\right)_{\mathbf{u}}{}^{\mathbf{v}}\mathfrak{g}_{\mathbf{vz}}+... \tag{3.5}$$

and may conclude that indeed, both the kinetic term for the graviphotons and the Yang-Mills vectors have to be present in the Lagrangian of the theory. Moreover, the coupling matrix of the kinetic term for the vectors

$$\Omega_{\mathbf{uz}} \;=\; \left(P^{\mathrm{SG}}-P^{\mathrm{YM}}\right)_{\mathbf{u}}{}^{\mathbf{v}}\mathfrak{g}_{\mathbf{vz}} \tag{3.6}$$

[1] The lowest component of this auxiliary superfield is the E^{ij} auxiliary field [19] in the component description of off-shell conformal $N=4$ supergravity.

is exactly the one appearing in the action exhibited in [6]. There it was argued that, in order for all vectors to be physical, the above coupling must be positive definite, which implies that the metric $\mathfrak{g}_{\mathbf{uz}}$ must have signature $(6,n)$. This additional information about the metric together with the equation (2.19) satisfied by the scalars allows to identify the sigma model $\frac{SO(6,n)}{SO(6)\times SO(n)}$ parameterized by the Yang-Mills scalars $\mathfrak{t}^{[\mathrm{BA}]\mathbf{z}}$. The identification of this sigma model first suggested in [5] and discussed also in [6] is detailed in appendix B.

On the other hand the imaginary part of the relation (3.4) gives the Bianchi Identity for the 2-form gauge field

$$\partial^a H^*_a \;=\; \frac{\mathrm{i}}{2} F^{*ba\mathbf{u}} F_{ba}{}^{\mathbf{z}} \left(P^{\mathrm{YM}} + P^{\mathrm{SG}}\right)_{\mathbf{u}}{}^{\mathbf{v}} \mathfrak{g}_{\mathbf{vz}} + \ldots \;=\; \frac{\mathrm{i}}{2} F^{*ba\mathbf{u}} F_{ba}{}^{\mathbf{z}} \mathfrak{g}_{\mathbf{uz}} + \ldots \tag{3.7}$$

The topological term $F^{*ba\mathbf{u}} F_{ba}{}^{\mathbf{z}} \mathfrak{g}_{\mathbf{uz}}$ is an indication of the intrinsic presence of Chern-Simons forms in the theory. Indeed, one needs just to explicit the projection $E^{\mathcal{A}}\| = e^{\mathcal{A}} = \mathrm{d}x^m e_m{}^{\mathcal{A}}$ on the 3-form H

$$\begin{aligned} H\| &= \frac{1}{2}\mathrm{d}x^m \mathrm{d}x^n \mathrm{d}x^k \partial_k b_{nm} \;=\; \frac{1}{3!} e^{\mathcal{A}} e^{\mathcal{B}} e^{\mathcal{C}} H_{\mathcal{CBA}}| \\ &= \frac{1}{3!} e^a e^b e^c H_{cba} + \frac{1}{2} e^a e^b \mathrm{d}x^m v_m{}^{\mathbf{u}} H_{\mathbf{u}ba} + \ldots \\ &= \frac{1}{3!} e^a e^b e^c H_{cba} + \frac{1}{2} e^a e^b \mathrm{d}x^m v_m{}^{\mathbf{u}} F_{ba}{}^{\mathbf{z}} \mathfrak{g}_{\mathbf{uz}} + \ldots \end{aligned} \tag{3.8}$$

where for the last line the relation (2.13) was used, in order to see that in the development of the supercovariant fieldstrength of the antisymmetric tensor $H^*_a|$ the fieldstrength $e_l{}^a \varepsilon^{knml} \partial_k b_{nm}$ is naturally accompanied by the Chern-Simons terms of both the graviphotons and the additional gauge vectors. This is an intrinsic property of soldering in central charge superspace, as pointed out in [12]. One can clearly see that it is the existence of an object $\mathfrak{g}$ in the central charge sector relating the 3–form components to those of the torsion by (2.13), which is responsible for this issue.

Recall, that in constructions of coupling of supergravity containing an antisymmetric tensor to Yang-Mills multiplets [14], the usual procedure is to define a modified fieldstrength for the antisymmetric tensor including by hand the Yang-Mills Chern-Simons terms in it. In this case, the gauge transformations of the Chern-Simons term are compensated by assigning suitably adjusted Yang-Mills gauge transformations to the antisymmetric tensor, and the modified fieldstrength is rendered invariant in this way. Let us verify, that this is automatical in our approach using central charge superspace. Here gauge transformations are identified as translations in the direction of the central charge coordinates. Indeed, taking the double bar projection [14] for the Wess-Zumino transformation of the frame component $E^{\mathbf{u}}$ along the vector field $\zeta^{\mathcal{A}} = (0,0,0,\zeta^{\mathbf{u}})$

$$\delta^{\mathrm{wz}}_{\zeta} E^{\mathbf{u}} \;=\; D\zeta^{\mathbf{u}} + \imath_{\zeta} T^{\mathbf{u}}, \tag{3.9}$$

one finds the usual transformation law for abelian gauge vectors

$$\delta_\zeta^{\rm wz} v_m{}^{\mathbf{u}} \;=\; \partial_m \zeta^{\mathbf{u}}. \tag{3.10}$$

However, writing the Wess-Zumino transformation for the 2–form gauge potential

$$\delta_\zeta^{\rm wz} B \;=\; \imath_\zeta H \;=\; \frac{1}{2} E^{\mathcal{A}} E^{\mathcal{B}} \zeta^{\mathbf{u}} H_{\mathbf{u}\mathcal{B}\mathcal{A}} \;=\; \frac{1}{2} E^{\mathcal{A}} E^{\mathcal{B}} \zeta^{\mathbf{u}} \mathfrak{g}_{\mathbf{uz}} T_{\mathcal{B}\mathcal{A}}{}^{\mathbf{z}} \;=\; \mathfrak{g}_{\mathbf{uz}} \zeta^{\mathbf{u}} T^{\mathbf{z}} \tag{3.11}$$

and taking its double bar projection one finds that the antisymmetric tensor transforms exactly into the fieldstrengths of the gauge vectors

$$\delta_\zeta^{\rm wz} b_{mn} \;=\; \mathfrak{g}_{\mathbf{uz}} \zeta^{\mathbf{u}} \left(\partial_m v_n{}^{\mathbf{z}} - \partial_n v_m{}^{\mathbf{z}} \right). \tag{3.12}$$

where $T^{\mathbf{z}}\| = \frac{1}{2} \mathrm{d}x^m \mathrm{d}x^n (\partial_n v_m{}^{\mathbf{z}} - \partial_m v_n{}^{\mathbf{z}})$ was used.

4. Conclusion and outlook

In this article we identified the N-T multiplet coupled to n abelian vector multiplets in the geometry of central charge superspace. Even though we started with $6+n$ gauge vectors, the geometry of the 3–form singled out the particular combinations of these which belong to the supergravity multiplet. The remaining independent combinations take part of the additional gauge multiplets. The supersymmetry transformations as well as main parts of some equations of motion are compared to the component formulations found by dimensional reduction [5] or using conformal methods [6]. In particular, we saw the emergence of the $SO(6,n)/SO(6)\times SO(n)$ sigma model for the Yang-Mills scalars as well as the presence of Chern-Simons terms in the supercovariant fieldstrength of the antisymmetric tensor or its particular transformation under Yang-Mills gauge transformations. On a more technical level we also could point out that the quadratic term in the gaugini sitting at the place of an auxiliary field of conformal supergravity was crucial.

Let us emphasize here that the features of the coupling pointed out in this article are very general properties of coupling supergravity containing an antisymmetric tensor with Yang-Mills gauge theory [21], currently used as guiding principles in superspace descriptions. In the articles [14] and [22] one can find a review of this kind of couplings in the four dimensional $N=1$ case. Also, an extensive list of references concerning various constructions of coupled systems of the same type in higher dimensions can be found in the same articles. Let us take for example [23]. There the aim was to incorporate string corrections up to first order in the string slope-parameter in the ten dimensional $N=1$ superspace. It turned out that the inclusion of both the Yang-Mills and Lorentz Chern-Simons terms in the geometry goes hand-in-hand with the presence of a source

$$A_{cba} \;\sim\; \beta' \mathrm{tr}(\lambda \sigma_{cba} \lambda) + \gamma' (T_{kl} \sigma_{cba} T^{kl}), \tag{4.1}$$

which appears in the double spinorial derivative of the dilaton and plays the role of an auxiliary field in the pure ten dimensional $N = 1$ supergravity. Observe, that A_{cba} is quadratic in the gaugino fields and the gravitino fieldstrength, and one can show in a similar way as we did above, that it is responsible for the curvature squared terms in the corresponding Lagrangian.

However, the interest of the central charge superspace approach applied here to the $N = 4$ case in four dimensions is that all the features of the Chapline–Manton coupling come out automatically offering us the possibility to just study the underlying mechanisms.

Considering possible generalisations of the work presented here, an obvious next step would be to check whether non-abelian vector multiplets can be described in this framework. Let us go back to the transformation law (3.9) of the frame component $E^{\mathbf{u}}$ in which the gauge vectors are identified and take its double projection in a more general setup

$$\delta_\zeta^{\mathrm{wz}} v_m{}^{\mathbf{u}} \;=\; \partial_m \zeta^{\mathbf{u}} \;+\; v_m{}^{\mathbf{v}} \zeta^{\mathbf{z}} T_{\mathbf{zv}}{}^{\mathbf{u}} \;+\; \ldots \tag{4.2}$$

Then the torsion component superfield $T_{\mathbf{zv}}{}^{\mathbf{u}}$ – vanishing in the present work – can play the role of structure constants and one can interpret the above equation as the transformation law for non-abelian gauge vectors. It would be interesting in particular to investigate whether superspace geometry implies some restrictions on the possible Lie groups.

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A. Supersymmetry transformations

Let us sum up in this appendix the supersymmetry transformations of the component fields. In superspace description of supergravity theories these are encoded in the formulas of Wess-Zumino transformations along a vector field ξ

$$\delta_\xi^{\mathrm{wz}} E^{\mathcal{A}} = \imath_\xi T + D\xi^{\mathcal{A}} \tag{A.1}$$

$$\delta_\xi^{\mathrm{wz}} B = \imath_\xi H \tag{A.2}$$

$$\delta_\xi^{\mathrm{wz}} \omega = \imath_\xi D\omega \tag{A.3}$$

where ω is a covariant superfield. Considering $\xi^{\mathcal{A}} = (0, \xi^\alpha_{\mathrm{A}}, 0, 0)$ one finds for the component fields of the supergravity multiplet

$$\delta_\xi^{\mathrm{wz}} e_m{}^a = \mathrm{i}\xi_{\mathrm{C}} \sigma^a \bar{\psi}_m{}^{\mathrm{C}} \tag{A.4}$$

$$\frac{1}{2}\delta_\xi^{\rm wz}\psi_{m}{}_{\rm A}^{\alpha} = \hat{\mathcal{D}}_m\xi_{\rm A}^\alpha - 2(\xi_{\rm C}\sigma_{ma})^\alpha U^{a{\rm C}}{}_{\rm A}| + \xi_{\rm C}^\alpha e_m{}^a\chi_a{}^{\rm C}{}_{\rm A}| + \frac{1}{8}\xi_{\rm A}^\alpha\left(\psi_{m{\rm B}}\chi^{\rm B} - \bar\psi_m{}^{\rm B}\chi_{\rm B}\right) - \frac{1}{8}(\xi_{\rm B}\chi^{\rm B})\psi_m{}_{\rm A}^\alpha \tag{A.5}$$

$$\frac{1}{2}\delta_\xi^{\rm wz}\bar\psi_{m}{}_{\dot\alpha}^{\rm A} = {\rm i}e_m{}^b(\xi_{\rm C}\sigma^a)_{\dot\alpha}F_{ab}^{{\rm SG}(-)\mathbf{z}}\mathfrak{t}^{[{\rm CA}]\mathbf{u}}\mathfrak{g}_{\mathbf{zu}}L^{-1/2} + \frac{q}{2}\psi_{m{\rm F}}\xi_{\rm C}\epsilon^{\rm FCAB}\bar\chi_{{\rm B}\dot\alpha} + \frac{1}{8}\bar\psi_m{}_{\dot\alpha}^{\rm A}(\xi_{\rm C}\chi^{\rm C}) - \frac{\rm i}{2}\left((\xi_{\rm B}\sigma_m\mathfrak{s}^{[{\rm A}\mathbf{z}})\mathfrak{s}_{\dot\alpha}^{{\rm B}]\mathbf{u}} + \frac{q}{2}\varepsilon^{\rm ABEF}(\xi_{\rm B}\mathfrak{s}_{\rm E}{}^{\mathbf{z}})(\mathfrak{s}_{\rm F}{}^{\mathbf{u}}\sigma_m)_{\dot\alpha}\right)\mathfrak{g}_{\mathbf{uz}} \tag{A.6}$$

$$\left(\delta_\xi^{\rm wz}v_m{}^{\mathbf{u}}\right)P^{\rm SG}{}_{\mathbf{u}}{}^{\mathbf{z}} = 2L^{1/2}\psi_{m{\rm B}}\xi_{\rm C}\mathfrak{t}^{[{\rm BC}]\mathbf{z}} - {\rm i}L^{1/2}(\xi_{\rm C}\sigma_m\bar\chi_{\rm B})\mathfrak{t}^{[{\rm CB}]\mathbf{z}} \tag{A.7}$$

$$\delta_\xi^{\rm wz}b_{mn} = 2{\rm i}L^{1/2}g_{\mathbf{uz}}v_{[n}{}^{\mathbf{u}}(\xi_{\rm C}\sigma_{m]})_{\dot\alpha}\left(\mathfrak{s}^{\dot\alpha{\rm C}\mathbf{z}} + \chi_{\rm F}^{\dot\alpha}\mathfrak{t}^{[{\rm FC}]\mathbf{z}}\right) + 2L\xi_{\rm C}\sigma_{mn}\chi^{\rm C} + 2{\rm i}L\bar\psi_{[n}^{\rm C}\sigma_{m]}\xi_{\rm C} + 4L^{1/2}v_{[m}{}^{\mathbf{u}}\psi_{n]{\rm F}}\xi_{\rm C}\mathfrak{h}_{\mathbf{u}}{}^{[{\rm FC}]} \tag{A.8}$$

$$\delta_\xi^{\rm wz}L = \xi_{\rm C}\chi^{\rm C}L \tag{A.9}$$

$$\delta_\xi^{\rm wz}\chi_\alpha^{\rm A} = -2(\sigma^{ab}\xi_{\rm C})_\alpha F_{ab}^{{\rm SG}\mathbf{z}}\mathfrak{t}^{[{\rm CA}]\mathbf{u}}\mathfrak{g}_{\mathbf{zu}} + \left(\xi_{\alpha{\rm C}}(\mathfrak{s}^{{\rm C}\mathbf{z}}\mathfrak{s}^{{\rm A}\mathbf{u}}) + q\varepsilon^{\rm CAEF}(\xi_{\rm C}\mathfrak{s}_{\rm E}{}^{\mathbf{u}})\mathfrak{s}_{\alpha{\rm F}}{}^{\mathbf{z}}\right)\mathfrak{g}_{\mathbf{zu}} - \frac{3}{4}(\xi_{\rm C}\chi^{\rm C})\chi_\alpha^{\rm A} \tag{A.10}$$

$$\delta_\xi^{\rm wz}\chi_{\rm A}^{\dot\alpha} = {\rm i}(\xi_{\rm A}\sigma^a\epsilon)^{\dot\alpha}\left(L^{-1}\mathcal{D}_aL - U_a - \chi_a\right)| + \frac{3}{4}(\xi_{\rm B}\chi^{\rm B})\chi_{\rm A}^{\dot\alpha} - (\xi_{\rm A}\chi^{\rm B})\chi_{\rm B}^{\dot\alpha} \tag{A.11}$$

For the components of the vector multiplets one obtains

$$\delta_\xi^{\rm wz}\mathfrak{t}^{[{\rm BA}]\mathbf{u}} = q\varepsilon^{\rm BADC}\xi_{\rm D}\mathfrak{s}_{\rm C}{}^{\mathbf{u}} \tag{A.12}$$

$$\delta_\xi^{\rm wz}\mathfrak{s}_{\alpha{\rm A}}{}^{\mathbf{u}} = (\sigma^{ab}\xi_{\rm A})_\alpha F_{ab}^{{\rm YM}\mathbf{u}}L^{-1/2} - 2(\xi_{\rm C}\mathfrak{s}_{\rm B}{}^{\mathbf{v}})\mathfrak{s}_{\alpha{\rm A}}{}^{\mathbf{z}}\mathfrak{t}^{[{\rm CB}]\mathbf{u}}\mathfrak{g}_{\mathbf{vz}} + \frac{1}{4}(\xi_{\rm B}\chi^{\rm B})\mathfrak{s}_{\alpha{\rm A}}{}^{\mathbf{u}} - \frac{1}{2}(\xi_{\rm A}\chi^{\rm B})\mathfrak{s}_{\alpha{\rm B}}{}^{\mathbf{u}} + \frac{1}{2}(\xi_{\rm A}\mathfrak{s}_{\rm B}{}^{\mathbf{u}})\chi_\alpha^{\rm B} + \xi_{\alpha{\rm B}}(\chi_{\rm A}\mathfrak{s}^{{\rm B}\mathbf{u}}) - \frac{1}{2}\xi_{\alpha{\rm A}}(\chi_{\rm B}\mathfrak{s}^{{\rm B}\mathbf{u}}) \tag{A.13}$$

$$\delta_\xi^{\rm wz}\mathfrak{s}^{\dot\alpha{\rm A}\mathbf{u}} = 2{\rm i}(\xi_{\rm B}\sigma^a\epsilon)^{\dot\alpha}\hat{\mathcal{D}}_at^{[{\rm BA}]\mathbf{u}} - 2(\xi_{\rm C}\mathfrak{s}_{\rm B}{}^{\mathbf{v}})\mathfrak{s}^{\dot\alpha{\rm A}\mathbf{z}}\mathfrak{t}^{[{\rm CB}]\mathbf{u}}\mathfrak{g}_{\mathbf{vz}} - \frac{1}{4}(\xi_{\rm B}\chi^{\rm B})\mathfrak{s}^{\dot\alpha{\rm A}\mathbf{u}} \tag{A.14}$$

$$\left(\delta_\xi^{\rm wz}v_m{}^{\mathbf{u}}\right)P^{\rm YM}{}_{\mathbf{u}}{}^{\mathbf{z}} = {\rm i}L^{1/2}\xi_{\rm C}\sigma_m\mathfrak{s}^{{\rm C}\mathbf{z}} \tag{A.15}$$

The fields $U_a{}^{\rm B}{}_{\rm A}$ and the traceless part of $\chi_a{}^{\rm B}{}_{\rm A}$ contain only quadratic terms in the spinors,

$$\widetilde{U}_a{}^{\rm B}{}_{\rm A} = \widetilde{\chi}_a{}^{\rm B}{}_{\rm A} = -\frac{\rm i}{8}\left(\widetilde{\chi^{\rm B}\sigma_a\chi_{\rm A}} + 2\mathfrak{g}_{\mathbf{zu}}\widetilde{\mathfrak{s}_{\rm A}{}^{\mathbf{z}}\sigma_a\mathfrak{s}^{{\rm B}\mathbf{u}}}\right) \tag{A.16}$$

$$U_a = \frac{\rm i}{8}\left(\chi^{\rm A}\sigma_a\chi_{\rm A} + 2\mathfrak{g}_{\mathbf{zu}}\mathfrak{s}_{\rm A}{}^{\mathbf{z}}\sigma_a\mathfrak{s}^{{\rm A}\mathbf{u}}\right) \tag{A.17}$$

while the trace part of $\chi_a{}^{\rm B}{}_{\rm A}$, corresponding to the $U(1)$ part of the initial $U(4)$ connection, contains the fieldstrengths of the antisymmetric tensor

$$\chi_a = {\rm i}H_a^*L^{-1} - \frac{3{\rm i}}{8}\left(\chi^{\rm A}\sigma_a\chi_{\rm A} + 2\mathfrak{g}_{\mathbf{zu}}\mathfrak{s}_{\rm A}{}^{\mathbf{z}}\sigma_a\mathfrak{s}^{{\rm A}\mathbf{u}}\right) \tag{A.18}$$

These expressions of the supersymmetry transformations can be compared to the component level results [5], [6], they are written automatically in terms of supercovariant fieldstrengths.

B. The $\frac{SO(6,n)}{SO(6)\times SO(n)}$ sigma model

All we know about scalars $\mathfrak{t}^{[\mathrm{BA}]\mathbf{u}}$ at dim 0 is that they satisfy the relations (2.19), implied by the Binachi Identities for the 3–form. The general solution of this equation can be written in a form

$$\mathfrak{t}^{[\mathrm{BA}]\mathbf{u}} \;=\; \mathfrak{t}^{(0)\,[\mathrm{BA}]\mathbf{z}}\mathcal{G}_{\mathbf{z}}{}^{\mathbf{u}} \tag{B.1}$$

where $\mathfrak{t}^{(0)\,[\mathrm{BA}]\mathbf{z}}$ is a particular solution and $\mathcal{G}$ is an element of the Lie group leaving the metric $\mathfrak{g}$ invariant. Since the signature of this metric was fixed to (6,n), this means that g is an element of $SO(6,n)$.

There is a particular solution of (2.19) which is already known and has a special meaning:

$$\mathfrak{t}^{(0)\,[\mathrm{BA}]\mathbf{z}} \;=\; \begin{pmatrix}\mathfrak{t}^{(0)\,[\mathrm{BA}]z}, & 0\end{pmatrix} \tag{B.2}$$

where the central charge indices $\mathbf{z}$ were splitted in two groups, $\mathbf{z} = (z,\bar{z})$, with $z = 1..6$ and $\bar{z} = 1..n$, and $\mathfrak{t}^{(0)\,[\mathrm{BA}]z}$ are the covariantly constant $\mathfrak{t}$ and $\mathfrak{h}$ matrices used in the pure N-T supergravity case [13]. In a similar way, one defines the matrix of constants

$$\mathfrak{q}_J^{(0)\mathbf{z}} \;=\; \begin{pmatrix}0, & \delta_J^{\bar{z}}\end{pmatrix} \tag{B.3}$$

with $J = 1..n$ and forms a $(6+n)\times(6+n)$ matrix as

$$S^{(0)} \;=\; \begin{pmatrix}\mathfrak{t}^{(0)\,[\mathrm{BA}]\mathbf{z}} \\ \mathfrak{q}_J^{(0)\mathbf{z}}\end{pmatrix}. \tag{B.4}$$

Now one can verify that $S^{(0)}$ is an element of $SO(6,n)$ and obviously,

$$S \;=\; S^{(0)}\mathcal{G} \;=\; \begin{pmatrix}\mathfrak{t}^{[\mathrm{BA}]\mathbf{z}} \\ \mathfrak{q}_J{}^{\mathbf{z}}\end{pmatrix}, \tag{B.5}$$

corresponding to a general solution is a general element of $SO(6,n)$. It is interesting to note that the particular solution $S^{(0)}$ corresponds to the "uncoupled" sugra+6YM system.

At this stage multiplications on the right by global elements of this group $G = SO(6,n)$ are well-defined

$$S \longrightarrow S\mathcal{G}, \qquad \mathcal{G}\in G. \tag{B.6}$$

The question is what subgroup K of $G = SO(6,n)$ can act on the left on S

$$S \longrightarrow \mathcal{K}^{-1}S, \qquad \mathcal{K}\in K \tag{B.7}$$

such that the corresponding gauge transformation is a symmetry of the theory. On the one hand, since only the scalar components $\mathfrak{t}^{[\mathrm{BA}]\mathbf{z}}$ appear explicitly, a local transformation

which leaves invariant the upper 6 rows of the matrix S in (B.5) is a symmetry of the action. This is an $SO(n)$ rotation having the representation

$$\mathcal{K}_{SO(n)} \;=\; \begin{pmatrix} \frac{1}{2}\delta^{\mathrm{BA}}_{\mathrm{DC}} & 0 \\ 0 & \mathcal{K}_J{}^I \end{pmatrix}. \tag{B.8}$$

On the other hand, the structure group of our superspace contains an $SU(4)$ factor which is automatically implemented as local symmetry of the theory. In particular, an $SU(4)$ transformation of a vector

$$u^{\mathrm{A}} \longrightarrow k^{-1\mathrm{A}}{}_{\mathrm{B}} u^{\mathrm{B}} \tag{B.9}$$

acts on the scalars S with the representation

$$\mathcal{K}_{SU(4)} \;=\; \begin{pmatrix} k^{\mathrm{B}}{}_{[\mathrm{D}} k^{\mathrm{A}}{}_{\mathrm{C}]} & 0 \\ 0 & \delta_J{}^I \end{pmatrix}. \tag{B.10}$$

In fact the constraint (2.18) insures that the $SU(4)$ connection $\hat{\Phi}^{\mathrm{A}}{}_{\mathrm{B}}$ is given as a function of the derivatives of the scalars,

$$\left(d\mathfrak{t}^{[\mathrm{BA}]\mathbf{u}} \right) \mathfrak{h}_{\mathbf{u}[\mathrm{DC}]} \;=\; 2\delta^{[\mathrm{B}}_{[\mathrm{D}} \hat{\Phi}^{\mathrm{A}]}{}_{\mathrm{C}]}. \tag{B.11}$$

This concludes our identification of the $\frac{SO(6,n)}{SO(6)\times SO(n)}$ sigma model parameterized by the scalars $\mathfrak{t}^{[\mathrm{BA}]\mathbf{u}}$ subject to the relation (2.19).

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